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Efficient Heuristic Algorithms for unweighted minimum vertex cover problem

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Abstract
Covering all edges in a graph with a small set of vertices is one of the most fundamental graph problems which is called the minimum vertex cover problem. In the literature different strategies have been employed to find near-optimal minimum vertex cover set in different kinds of graphs.
In this work, two efficient algorithms (i.e., MAxA and MAxAR) are introduced to find the minimum vertex cover set in any unweighted undirected graph. The proposed construction algorithms have two main steps in each iteration which explore neighborhoods of minimum degree vertices to find and select appropriate vertices for the cover set. Until all of the edges are removed or selected in the algorithms, these two steps are performed iteratively. The proposed algorithms have been implemented on DIMACS, BHOSLIB, and other benchmarks where experimental results show that the proposed algorithms outperform other relevant methods in terms of time and cardinality of vertex cover set.
Keywords: NP problem, Minimum vertex cover, Heuristic algorithm.

1. Introduction

There are many reasons for using surveillance cameras on city roads. In a road graph, the edges and nodes represent the roads and intersections, respectively. What we are required to do is to put cameras in the intersections to see the city in its entirety. In addition, as few cameras as possible should be used for cost efficiency. For a simple graph G, C is a subset of vertices such that at least one ends of each edge is in C. Therefore, the goal is to find minimum vertex cover in the road graph. Below, some of real-world applications for minimum vertex cover are listed [1]:

In a wireless sensor network, the minimum vertex cover can be used for monitoring links, routing, data aggregation, clustering [2, 3], and so on. For example, in reference [4], the authors have proposed algorithms, which are used to increase the lifespan of a wireless sensor network. By selecting a minimum number of nodes, the mentioned algorithms reduce energy consumption.

In biochemistry calculations [5], the problem of minimum vertex cover has been used for the structure of the phylogenetic tree as well as for analyzing multiple sequence levels to infer the evolutionary relationships between genes and proteins. In the conflict graph, in which the vertices and edges represent the sequence of samples and the conflict between the corresponding sequences respectively, the minimum vertex cover is used to eliminate all of the paradoxes by removing the smallest sequence.

To protect large networks against propagation of hidden worms in real-time, the minimum vertex cover has recently been used in a graph in which the vertices and edges are the routing servers and connections between routing servers, respectively [6].

In other computer fields such as scheduling, VLSI design, and signal transmission, the minimum vertex cover is used to find the closest solution to the perfect solution [7]. An efficient algorithm for minimum vertex cover can improve parameters of time, cost, and resource allocation in different applications.

The potential applications of minimum vertex cover reveal its significance. In addition, due to the NP nature of the problem, it still draws the attention of researchers. Several exact algorithms [8-11] such as Branch and Bound, LP-Based Branch and Cut as well as approximate algorithms have been proposed to solve the problem. On the other hand, to find the optimal solution for the minimum vertex cover, any factor smaller than 1.3606 [12] is considered an NP-hard problem. Many algorithms have been proposed for the minimum vertex cover problem, including the maximum degree greedy algorithm (MDG) [13], vertex support algorithm (VSA) [14], modified vertex support algorithm (MVSA) [15], and the maximum degree adjacent to the minimum degree algorithm (MAMA) [16]. The construction algorithm for minimum vertex cover problem is based on finding a solution through expanding a partial vertex set. Primarily, in these algorithms, the vertex set is empty. Then, until it becomes a cover set, the vertices are repeatedly added to the set. MDG and VSA are greedy algorithms. Each one of them selects the maximum vertex degree and maximum vertex support in each period and then updates the graph. MVSA is a modified version of VSA. MAMA selects the vertex with the maximum degree in the neighborhood of the vertex with the minimum degree in each round.

The proposed algorithms similar to MAMA search the neighborhood of minimum degree vertices. However, in the proposed algorithms for each minimum degree vertices, one of the neighbors is selected in each period. The effectiveness of our proposed algorithms has been observed on the benchmark of small and large graphs. The simulation results have also shown that our proposed algorithms find the vertex cover set with a smaller number of vertices. In addition, it takes less time compared to MAMA.

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The rest of the paper is organized as follows. Section 2 briefly addresses a selected number of more efficient MVC algorithms. In Section 3, the proposed algorithms have been introduced while experimental evaluations are described in Section 4. Finally, we draw our conclusions in Section 5.

2. Review of Literature

Because finding the best solution to the problem is impossible. This problem can be studied to find better solutions in the future. Several new methods, such as quantum computers, are now used to find the appropriate solution to the MVC problem [42]. But there are two challenges to mention:

1. As technology advances, the amount of data also increases. MVC is one of the few problems where kernelization (reducing input size by iterative data reduction rule) is known to be really very efficient. As a result, data management and big data issues can become important in the field. 2. The above MVC solutions use the local search method. Therefore, it is necessary to find different solutions to avoid getting stuck in local optimization. These solutions should also be pretty fast. In other words, it will find several suboptimal solutions in a short time and choose the best one among them. Every day, the application of the MVC problem manifests itself more in all types of networks, such as recently the problem was used to monitor surveillance cameras in the transport network in Russia [43]. In addition, in a paper made to predict and diagnose COVID-19; Graph theory and MVC were used to provide a suitable solution [44]. In the past six decades, a number of methods have been proposed to solve the minimum vertex cover. Most of these methods are approximate solutions. In reference [17], Anton investigated the difficulty of an algorithm on a graph that is e-dense everywhere; he showed that if we want to solve MVC problem with a factor less than $\frac{7+\varepsilon}{6+2\varepsilon}$, it turns into an NP-hard problem. In the present section, we review the algorithms for the minimum vertex cover with polynomial time complexity. Evolutionary [18 –22] and local search [23-35] algorithms are the two most common approaches for solving minimum vertex cover problem.

The most popular heuristic method for NP-hard combinatorial optimization problems is local search where, numerous important algorithms of this type are applicable to complicated problems such as satisfiability, coloring, and routing problems. Cai et al. proposed NuMVC, FastVC, NuMVC2+p, and FastVC2+p which have desirable performance in massive graphs. NuMVC includes two-stage exchange and uses "edge weighting with forgetting" to deal with and fix the problems of local search methods. In FastVC, using two heuristic functions leads to less time complexity and better performance. Improving the previous algorithms on the one hand and adding a pre-processing step to them, on the other hand, lead to NuMVC2+p and Fast VC2+p algorithms in finding the minimum vertex cover for large sparse graphs.

Several heuristic algorithms including 'Evolutionary Algorithm', 'Simulated Annealing', and 'Branch and Bound' algorithms are used to solve the problem of minimum vertex cover in a random graph that is modeled with $G(n, c/n)$. The efficiency of these algorithms has been investigated for different values of c [36, 37].

Time complexity of MDG is $O(E^2)$. In the VSA, there is an adjacency matrix that is used to calculate the sum of degrees for all vertices. This value is called vertex support. Afterwards, a vertex with the maximum support is added to the cover set in each period. This takes $O(V^2)$.

MVSA calculates the vertex support in the same way as VSA. Then, in each period, it finds the vertices with minimum support and then adds the minimum neighbor to the cover set. The time complexity of MVSA is $O(EV^2 \log V^2)$. MAMA adds the largest neighbor of the minimum degree vertices in each period to the cover set. The time complexity of MAMA equals to $O(EV^2)$.

3. Proposed Algorithm

In this section, two algorithms are proposed to solve the minimum vertex cover problem, both of which are based on the neighbors of minimum degree vertices. The goal is to select nodes that can cover the largest number of uncovered edges. Considering MDG and MAMA algorithms in exploring the minimum degree neighbors, MaxA and MaxAR algorithms are introduced here where the time complexity of both algorithms is $O(EV^2)$. Primarily, the algorithms start with an empty cover set, and two steps are repeated to cover all edges of the graph. The first step in both algorithms is to find the vertices with minimum degree (i.e., Min set). In the second step, the neighbors of the Min set are searched to find the best candidates.

In MaxA, for each vertex in the Min set, its largest neighbor (the neighbor with the largest degree) is selected and added to the cover set.

MaxAR, which is an extension of MaxA, selects the best candidate for each vertex in the Min set between a maximum degree vertex and a random vertex among neighbors. The probability of choosing between each of these two choices is equal to 0.5. In fact, MaxAR is a combination of MaxA algorithm and random walk (RW) algorithm. RW is a preferable technique for heuristic approaches because it is simple to use and it can easily escape complex boundaries and local optima and cause extensive exploration in the state space. As mentioned earlier, MVC is an NP-hard problem and it needs effective heuristics to achieve admissible performance. The evaluation of the proposed algorithm shows that this goal is achieved with the help of the RW algorithm. Based on these observations, we have presented our two algorithms below, note that here Z refers to node(s) with no edges:

Algorithm1:

Algorithm MaxA: Input $G(V,E)$, Output : Cover

1. Cover = {}
2. While G is not empty
3. Z = node(s) with degree 0
4. Remove Z from $G(V,E)$
5. Min = node(s) with minimum degree
6. $Max_{adjacent}$= find a maximum degree adjacent to each node in Min
7. Cover = Cover ∪ $Max_{adjacent}$
8. Delete Min and its vertices
9. End while

Algorithm 2:

Algorithm MaxAR: Input: $G(V,E)$, Output : Cover

1. Cover = {}
2. While G is not empty
3. Z= node(s) with degree 0
4. Remove Z from $G(V,E)$
5. Min = node(s) with minimum degree
6. $Max_{adjacent}$= find a maximum degree adjacent to each node in Min
7. $Random$ = Select a random adjacent to each node in Min
8. $Selected$= for each node in Min select one vertex between its $Max_{adjacent}$ and $Random$ with probability ½.
9. Cover = Cover ∪ $Selected$
10. Delete Selected
11. End while

In Table 1, we have checked the time complexity of MAMA and MDG algorithms and our two proposed algorithms. As it is evident, the time complexity of our algorithms and MAMA are equal.

Table1. Time complexity of simulated algorithms

MDG	MAMA	MaxA	MaxAR
$O(E^2)$	$O(EV^2)$	$O(EV^2)$	$O(EV^2)$

4. Experimental Results

A series of experiments were conducted to evaluate the performance of our proposed algorithms and compared their results with MAMA and MDG. The experimental results in [16] showed that MAMA has better performance in comparison to VSA, MDG, and MVSA in the benchmarks of small and large graphs. Therefore, if we show the superiority of the proposed algorithms over MAMA, their superiority over MDG, VSA, and MVSA methods is also confirmed.

In our research, algorithms were coded in Pycharm 2020.3.1 x64. Simulations were also performed on an Intel core i5 computer with a Windows 10 operating system.

Tables 2 and 3 show the efficiency of the proposed algorithms in different benchmarks. DIAMCS and BHOSLIB are the machine-learning benchmarks mentioned in the related articles. Other datasets such as biological networks, social networks, web graphs, technological networks, etc. [39] are related to different networks. The results of the DIAMCS [40] and BHOSLIB [41] benchmarks are compiled in Table 2 and the results of other benchmarks are shown in Table 3. Here, C^* and $|V|$ refer to the optimal solutions and the number of vertices, respectively. As shown in Tables 2 and 3, our proposed algorithms outperform MAMA, and thus outperform the others.

Table 2. The results of applying MDG, MAMA, and proposed algorithms on DIAMCS and BHOSLIB

Benchmark	$\|V\|$	C^*	Cardinality of the vertex cover				Approximation ratio			
			MDG	MAMA	MaxA	MaxAR	MDG	MAMA	MaxA	MaxAR
Brock200-1	200	187	188	196	188	**187**	1.005	1.048	1.005	1.000
Brock200-4	200	185	187	189	**185**	**185**	1.010	1.021	1.000	1.000
C125	125	114	115	121	**114**	114.7	1.008	1.061	1.000	1.006
C250.9	250	245	245	259	246	246	1.000	1.050	1.004	1.004
C500.9	500	485.4	487	488	486	**485.4**	1.003	1.005	1.001	1.000
c-fat200-1	200	128	139	180	**128**	**128**	1.080	1.406	1.000	1.000
c-fat200-2	200	144	145	154	**144**	144.2	1.006	1.069	1.000	1.001
c-fat500-1	200	341	345	352	**341**	**341**	1.011	1.032	1.000	1.000
c-fat500-10	500	372	372	376	**372**	373	1.000	1.010	1.000	1.002
c-fat500-2	500	361	363	372	**361**	361.4	1.005	1.030	1.000	1.001
c-fat500-5	500	369	370	391	385	**369**	1.002	1.059	1.043	1.000
Dsjc-500	500	488.9	491	495	490	**488.9**	1.004	1.012	1.002	1.000
Fbr-30-15-2	450	423	423	432	429	429.5	1.000	1.021	1.014	1.015
Fbr35-17-2	595	567	568	574	**567**	**567**	1.001	1.012	1.000	1.000
Hamming10-2	1024	966	966	1007	982	982	1.000	1.042	1.016	1.016
Hamming6-2	64	50	50	53	53	52	1.000	1.06	1.060	1.040
Hamming6-4	64	46	47	50	**46**	46.1	1.020	1.086	1.000	1.002
Hamming8-2	256	230	230	250	234	233	1.000	1.086	1.017	1.013
Hamming8-4	256	230	230	246	234	234.9	1.000	1.069	1.017	1.021
Johnson16-2-4	120	104	104	109	107	107	1.000	1.048	1.028	1.028
Johnson32-2-4	496	463	463	475	469	469.6	1.000	1.025	1.012	1.014
Johnson8-2-4	28	19	20	24	**19**	**19**	1.052	1.263	1.000	1.000
Johnson8-4-4	70	58	58	70	**58**	58.1	1.000	1.206	1.000	1.001
Keller4	171	151	151	158	**151**	**151**	1.000	1.046	1.000	1.000
Keller5	776	741	745	750	742	**741**	1.005	1.012	1.001	1.000

phat300-1	300	262.7	264	273	264	**262.7**	1.004	1.039	1.004	1.000
phat300-2	300	273	275	278	**273**	274	1.007	1.018	1.000	1.003
phat300-3	300	292	293	293	**292**	**292**	1.003	1.003	1.000	1.000
phat700-1	700	640.9	641	645	642	**640.9**	1.000	1.017	1.004	1.000
phat700-2	700	654	654	672	656	655.1	1.000	1.027	1.003	1.001
phat700-3	700	692	693	705	**692**	692.3	1.001	1.018	1.000	1.000
Sanr200-0.7	200	186	187	191	**186**	186.2	1.005	1.026	1.000	1.001
Sanr200-0.9	200	187	187	190	**187**	**187**	1.000	1.016	1.000	1.000
Sanr400-0.5	400	378	382	385	379	**378**	1.010	1.018	1.002	1.000
Sanr400-0.7	400	384	386	391	**384**	**384**	1.005	1.018	1.000	1.000

Table 3. The results of applying MDG, MAMA, and proposed algorithms on benchmark instances [39]

Benchmark	$\|V\|$	C^*	**Cardinality of the vertex cover**				**Approximation ratio**			
			MDG	**MAMA**	**MaxA**	**MaxAR**	**MDG**	**MAMA**	**MaxA**	**MaxAR**
bio-celegans	453	255	259	285	**255**	255.2	1.015	1.117	1.000	1.000
bio-diseasome	516	285	285	371	**285**	**285**	1.000	1.301	1.000	1.000
bio-dmela	7393	2658	2666	2968	**2658**	2663.8	1.003	1.116	1.000	1.002
bio-yeast	1458	456	463	557	**456**	457	1.015	1.221	1.000	1.002
ca-CSphd	1882	550	556	739	**550**	551	1.010	1.343	1.000	1.001
ca-Erdos992	6100	461	461	474	**461**	**461**	1.000	1.028	1.000	1.000
ca-GrQc	4158	2210.3	2219	2719	2211	**2210.3**	1.003	1.023	1.000	1.000
ca-HepPh	11204	6557.6	6574	٧٦١٦	٦٥٥٨	**6557.6**	1.002	1.161	1.000	1.000
ca-netscience	379	214	214	275	**214**	**214**	1.000	1.285	1.000	1.000
Frb30-15-1	450	424	429	424	425	424.7	1.011	1.000	1.002	1.001
Frb30-15-2	450	425	431	426	**425**	**425**	1.014	1.002	1.000	1.000
Frb30-15-3	450	425	429	425	425	425.6	1.009	1.000	1.000	1.001
ia-email-EU	32430	820	820	٨٧٤	**٨٢٠**	**820**	1.000	1.065	1.000	1.000
ia-email-univ	1133	604.2	608	٦١٨	٦٠٥	**604.2**	1.006	1.022	1.001	1.000
ia-enron-only	143	87	87	٨٨	٩١	90.6	1.000	1.011	1.045	1.041
ia-fb-essages	1266	589.4	595	٥٩٣	٥٩٠	**589.4**	1.009	1.006	1.001	1.000
ia-infect-dublin	410	297	297	٣٠٥	٢٩٨	298.2	1.000	1.026	1.003	1.004
ia-infect-hyper	113	92	93	٩٣	**٩٢**	**92**	1.010	1.010	1.000	1.000
ia-reality	6809	81	81	٨١	٨١	81	1.000	1.000	1.000	1.000
inf-power	4941	2215	2275	٢٤٩٨	**٢٢١٥**	2221.8	1.027	1.127	1.000	1.003
scc_enron-only	151	138	139	١٣٨	١٣٨	138	1.007	1.000	1.000	1.000
scc_fb-forum	897	372	372	٣٧٣	٣٧٢	372	1.000	1.000	1.000	1.000
scc_infect-hyper	113	110	110	١١٠	١١٠	110	1.000	1.000	1.000	1.000
scc_retweet	18469	562	562	٥٩٦	٥٦٣	563.3	1.000	1.060	1.001	1.002
scc_rt_lolgop	9742	103	103	١١٤	**١٠٣**	**103**	1.000	1.106	1.000	1.000
tech-routers-rf	2113	796	805	939	٧٩٦	**796**	1.011	1.179	1.000	1.000
tech-WHOIS	7476	2288	2298	٢٦٥٣	**٢٢٨٨**	2288.7	1.004	1.159	1.000	1.000
web-erkStan	12305	5492	5492	٦٧٦٤	٥٦٦٨	5640.3	1.000	1.231	1.032	1.027
web-edu	3031	1451	1587	١٥٨٤	**١٤٥١**	**1451**	1.093	1.091	1.000	1.000
web-indochina-2004	11358	7300	7424	٨٢٢٤	**٧٣٠٠**	**7300**	1.016	1.126	1.000	1.000
web-spam	4767	2321	2346	٢٤٩٧	**٢٣٢١**	2322.6	1.010	1.075	1.000	1.000
web-webbase-2001	16062	2667	2687	٣٣٤٣	**٢٦٦٧**	2668	1.007	1.253	1.000	1.000

As shown in Table 3, the proposed algorithms performed better on the same benchmarks, but in general both algorithms showed better coverage rates on sparse graphs compared to the other methods where in dense graphs, their response is often the same as the other methods. The average and the best approximation ratio for MAMA, MDG, and our proposed algorithm are shown in Table 4 where the approximation ratio is equal to $\rho = \frac{\text{Cardinality of the vertex cover}}{C^*}$. The superiority of the proposed algorithms is evident from the ratios in Table 4.

Table 4. Comparison of approximation ratio based on experimental result

Algorithms	**Worst ρ**	**Average ρ**
MDG	1.093	1.040
MAMA	1.343	1.098
MaxA	1.045	1.002
MaxAR	1.041	1.002

In most of the benchmarks instances, MAMA, MaxA, and MaxAR algorithms take the same execution time. However, experimental results have shown that in seven benchmarks, MAMA required more time than other algorithms to run. These results are shown in Table 5.

Table 5. Comparison of execution time

Benchmark	**MDG $\|ms\|$**	**MAMA $\|ms\|$**	**MaxA $\|ms\|$**	**MaxAR $\|ms\|$**
tech-routers-rf	0.48	15.89	0.73	0.81
tech-WHOIS	9.46	1140.76	18.36	18.73
web-BerkStan	25.38	271.34	25.65	**24.34**
web-edu	1.44	7.95	1.29	**1.12**
web-indochina-2004	49.06	3131.18	87.17	89.13
web-spam	7.34	611.51	17.33	16.16
web-webbase-2001	13.47	1168.00	27.35	24.73

5. Conclusion

The well-known minimum vertex cover problem cannot be solved in polynomial time. In this article, two heuristic algorithms MaxA and MaxAR were introduced both of which can be used for any large and small graph. The results revealed that the proposed algorithms had better performance in sparse graphs, that is, as the number of vertices increased, the efficiency of the proposed algorithms became more apparent. It is expected that the proposed algorithms will be more efficient in large sparse graphs.

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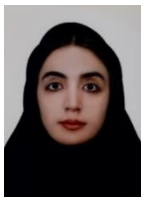

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